“Why Phi”

a derivation of the Golden Mean ratio based on heterodyne phase conjugation

(rewrite from original concept “Why Phi” of 2003)

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1. Abstract

The optimisation of phase-conjugation for a scale-invariant heterodyne wave packet is analysed. The principle of scale-invariance is introduced anticipating its application in soliton wave physics and could be a requirement for optimal phase-conjugation. Heterodyne phase-conjugation here is limited to the rudimental form of one wavelength being the sum or difference of two others. A numerical demonstration of the optimal solution, a Golden Mean scaling ratio, is given by means of a heterodyne power scan over a wide range of ratio’s. The Fourier spectrum of a Golden Mean recursive heterodyne is given, which was found to have the form of a binary Fibonacci series of frequencies. The function of heterodyne phase-conjugation in soliton creation will briefly be discussed.

2. Introduction

The “Golden Mean” ratio, also called “Phi” (Φ) barely needs an introduction. Not only has the Golden Mean section been used by artists (painters, sculptors, architects) since antiquity as a measure of beauty and harmony - the presence of Phi in Nature is just as ubiquitous, be it often in hidden form. At this point the famous Fibonacci series needs to be mentioned, $F_{n+1} = F_n + F_{n-1}$, which for greater values of n approaches $F_n / F_{n-1} = \Phi$. It seems as if Nature, where she wants to use Phi for reasons which may follow from this paper, but has only integer values to play with, chooses Fibonacci numbers instead. The sheer endless specialties of Phi, which is sometimes described as the “most irrational number”, have exhaustively been explored in mathematics.

Despite its ubiquity in practical applications, the Golden Mean ratio was never derived based upon fundamental physics criteria. However, the field of “Sacred Geometry”, which we might consider intuitional science and where Phi is also prominently present, is being disseminated by one of its leading proponents, Dan Winter[1], as an actual modelling of sustainable (mostly spherical) wave patterns in 3-D space. This insight has been a major source of inspiration for this paper.

3. Derivation of Phi from a scale-invariant heterodyne set

Heterodyne phase conjugation is defined as:

(1) $$\lambda_n = \lambda_{n+1} + \lambda_{n+2}$$

This describes the property of heterodyning that wavelengths add up and subtract.

Scale invariance can be defined as an arbitrary scaling ratio f:

(2) $$\frac{\lambda_n}{\lambda_{n+1}} = \frac{\lambda_{n+1}}{\lambda_{n+2}} = f$$

Rewriting (2) gives:

$$\lambda_n = \frac{\lambda_{n+1}^{2}}{\lambda_{n+2}}$$

Plugging into (1) gives:

$$\frac{1}{\lambda_{n+2}} \cdot \lambda_{n+1}^{2} - \lambda_{n+1} - \lambda_{n+2} = 0$$

The roots can be found using $$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

giving $$\lambda_{n+1} = \frac{1 \pm \sqrt{(1 + 4)}}{2 / (\lambda_{n+2})}$$

so that

(3) $$\lambda_{n+1} = 1.618034 \cdot \lambda_{n+2} \quad (\text{or} \quad \lambda_{n+1} = -0.618034 \cdot \lambda_{n+2}) \quad \text{and}$$

(4) scaling ratio f: $$\frac{\lambda_{n+1}}{\lambda_{n+2}} = 1.618034 = \Phi$$

4. A numerical proof of integral wave power of a scale-invariant phase-conjugate heterodyne optimized by Φ.

A composite scale-invariant heterodyne is created using a software program written by the author, "Fractal Synthesizer" which performs an integral wave power scan over a range of ratios. The result is a graph ranging from ratio 1.2 to 2.1 in steps of 0.001 for an adjustable number of heterodynes. The index "Hz" is used only for convenience, and indicates precisely one wave period in the display. The "power graph" has many interesting features but is used here primarily to illustrate the principle.

The composed scale-invariant heterodyne is given by:

$$E_H = \sum_{a=0}^{n} \left\{ \prod_{e=1}^{h} \cos\left(a \cdot da \cdot r^{e}\right) \right\} \qquad (5)$$

where:

E_H = Total energy of heterodyne set for a specific ratio – this is plotted
a = phase sample count
n = number of phase intervals
da = phase interval
∏ = Total energy at each step – (note that heterodyning is done by multiplying amplitudes)
e = exponent for scaling ratio
h = number of heterodyne harmonics
r = ratio - note: (5) gives the integral wave power for one specific ratio r only; when (5) is used to created the power-plot over a range of ratio's, the software steps up the ratio and plots $E_H = f(r)$

Below graph is the "power scan" $E_H = f(r)$ of a 10 "Hz" signal with one lower and 2 higher harmonics, for r = 1.2 to 2.1, with Δr = 0.001. Frequency settings are chosen to result in a representative graph, given the available FFT- and display resolution. Generally, the more harmonics are included, the narrower the peaks become, until the composed heterodyne virtually becomes a singularity.

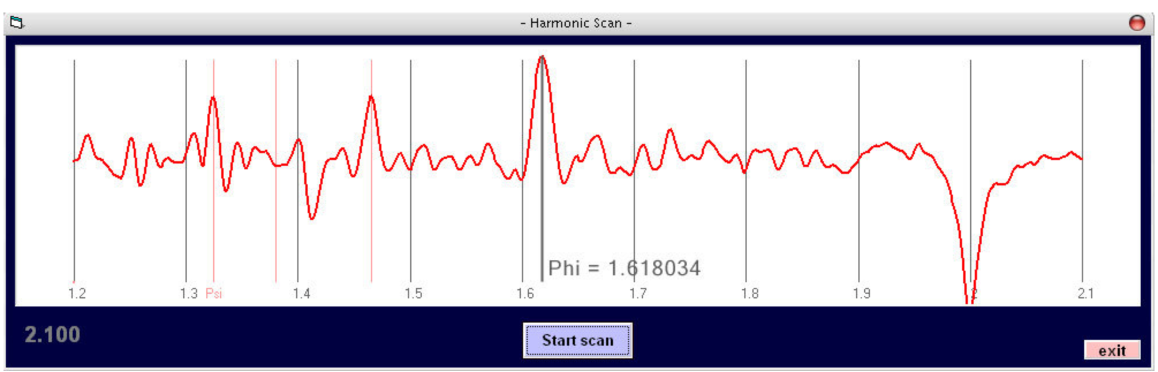

Fig. 1 – "Power scan" $E_H = f(r)$

The Golden Mean ratio (1.618..) clearly stands out as a recursive heterodyne ratio with maximum energy conservation through optimized phase-conjugation. A "family" of other, but less optimizing ratio's can be found e.g. by solving iteratively:

$$R-1 = \frac{1}{R^e} \tag{6}$$

Resulting in the following ratios for a few values of e:

Exponent (e)	**Ratio (R)**
0	2.000 (octave)
1	1.618 (Φ)
2	1.466
3	1.380
4	1.325 (Psi or "Plastic Ratio")
5	etc..

table 2 – Some special ratios, according to (6)

Other special ratio's include for example: $\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt{3}$, $\sqrt[3]{3}$ and the Tribonacci constant, 1.839... Especially the ratio 2 (octave) negative peak here is recognized by what might at first sight look like destructive phase-conjugation, however is believed to be important for soliton physics.

5. Fourier analysis of the Golden Mean scale-invariant heterodyne.

A composite Golden Mean based recursive heterodyne was spectrum analysed using the Fast Fourier Transform[3]. The result was found to be a series of frequencies with spacings which can be registered by the so called "Binary Fibonacci" series, also known as the "Golden String" or "Golden Sequence". This is a fractal string. The Golden Mean heterodyne spectrum below was created with the "Fractal Synthesizer" software. This is included to further illustrate the uniqueness of Phi in heterodyne phase conjugation.

Fibonacci nr.	**Decimal Fibo.**	**Binary Fibonacci (frtl)**
1	1	1
2	1	01
3	2	101
4	3	01101
3	5	10101101
6	8	0110110101101
7	13	101011010110110101101

table 2 – Decimal and binary Fibonacci numbers

Below graph shows the “Golden Sequence” fractal string, where the two different spacings in the frequency series are seen. This is the spectrum of a 14 “Hz” base frequency with 4 higher and 2 lower Phi-harmonics. Frequencies are chosen, again, to produce a representative graph given the FFT- and display resolution.

The code should be read from the right to the left, as follows:

> **ΔF_{max}** is the wider distance between two frequencies, indicated as “1”
> **ΔF_{min}** is the smaller distance, indicated as “0”

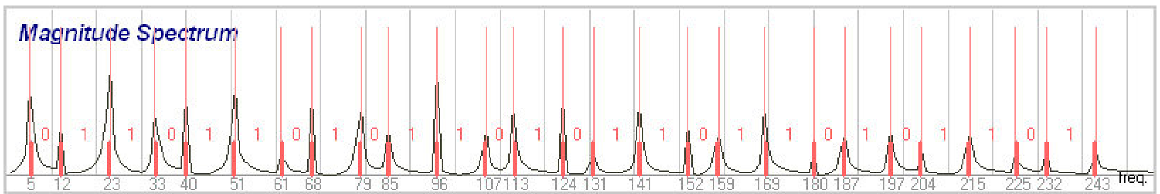

Fig. 2 – Perfect “Golden String” distributed spectrum when heterodyne scaling ratio is precisely Φ (1.618..)

The ratio between the two distances is obviously:

$$\Delta F_{min} : \Delta F_{max} = 1 : \Phi \qquad (7)$$

as a result of the fractally embedded, Φ - recursive heterodynes. The software also algorithmically calculates an ideal “Golden String” derived from the frequency settings alone, which can be used to verify the peaks found in the Fourier spectrum. In this graph they match perfectly. Some main “Golden String” parameters are algorithmically defined as per below table. Based on the known (heterodyne) frequency settings, the software then tries to make the closest match for each subsequent peak found.

number of heterodynes	**Lowest spectrum freq.:**	**ΔF_{min}**	**ΔF_{max}**
{0, 3, 6…}or n mod 3 = 0	$F_{S,0} = 2 \times F_{H,0}$	$F_{S,0} / \Phi$	$F_{S,0}$
{1, 4, 7…}or n mod 3 = 1	$F_{S,0} = F_{H,0}$	$F_{S,0} \times 2 / \Phi$	$F_{S,0} \times 2$
{2, 5, 8…}or n mod 3 = 2	$F_{S,0} = F_{H,0} / \Phi$	$F_{S,0} \times 2$	$F_{S,0} \times 2 \times \Phi$

table 3 – “golden string” spectrum parameters

If the ratio is changed slightly off-Phi, the coherent, fractal line-up is quickly destroyed. Here the ratio is 1.628, that is only 0.6% off Φ and the Golden Sequence cannot be found:

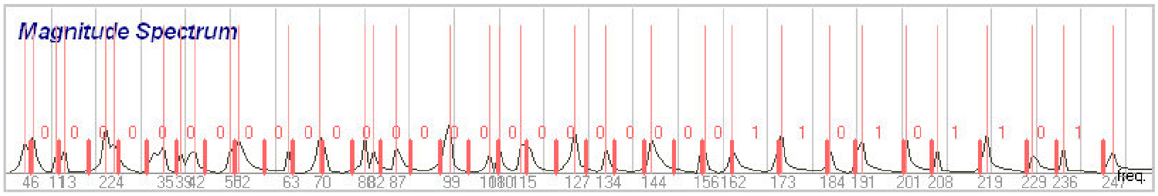

Fig. 2 – Distorted spectrum when heterodyne scaling ratio is slightly off - Φ (1.628..)

6. A definition of the term "phase-conjugation"

It is proposed that the term "phase-conjugation" be used only for heterodyning, that is, for wave physics in a non-linear medium (e.g. nonlinear optics) or in a more general sense defined by actual phase coherence, as described above. Likewise, the term "interference" should be used for "normal" (Newtonian, additive) wave interactions only and not for heterodyning.

7. The role of heterodyne phase conjugation in soliton creation

A "soliton" is a name for a "self-reinforcing" singular wave in a (weakly) non-linear, or non-linearly acting medium. It is considered sucking energy from its surrounding waves in a way for which there is no linear explanation. This accounts for its unusual permanence. As may follow from the discussion of above power-scan (graph 1), a highly harmonically inclusive, phase-conjugate heterodyne is virtually undetectable, and thus indestructible, but nevertheless transports energy and can therefore have a significant physical effect in a non-linear (e.g. meta-stable) medium. It is conjectured that:

1st. Scale-invariance is the only way to optimize phase-conjugation among many heterodynes;
2nd. Scale-invariance is the only way to stabilize phase-conjugation in a 3-dimensional medium, and
3rd. Scale-invariance is therefore the only way to distribute and store energy non-linearly - this is a requirement for phase-conjugate heterodyning to have a relevant physical effect in a non-linear medium (material or other), in principal form as a soliton.

Additional note: a soliton is commonly known as a solitary material wave, for example the epic, but rather infamous oceanic "rogue waves"[4]. However the concept can be generalized to include all sorts of solitary phenomena, from all normal waves taken as a whole (sound, electro-magnetic, atmospheric), vortex and toroidal waves, photons, atoms, biological cells, organisms, ecosystems and galaxies, in their respective arenas. It is hypothesized that the essential system dynamics is the same in all cases and includes a substantial heterodyne phase-conjugate aspect, in various ways coherently interacting with the purely material form and imparting it with a touch of vitality beyond standard explanation. A specialty of this approach to soliton creation lies in the distinct roles of the most prominent phase-conjugate nuclei, controlled by Phi and octave ratio (see power scan, fig. 1) with resp. an energy distributing and -assimilation function, relative to the average background level. This predicts the precise interactions with resp. thermo-dynamic and inertial environments, which however goes beyond the scope of this paper.

8. Conclusions

It was mathematically proven that the Golden Mean (Phi) ratio perfects phase-conjugation in a scale-invariant heterodyne wave packet, in the ideal case forming an infinite Phi-recursive series. Also it was numerically demonstrated that Phi ratio maximizes the integral wave power of this type of heterodyne (using a limited series). Further it was shown that the Fourier spectrum of a (limited) Golden Mean recursive heterodyne forms a binary Fibonacci spaced series of frequencies. This shows that Phi could be a unique mediator between heterodyne and normal physics. The physical relevance of this type of heterodyne wave packet is supported by the abundance of Fibonacci series in nature. The term "phase-conjugation" has been put in perspective and is proposed to be used for heterodyne processes only. A more generalized "soliton" concept was proposed, which is intrinsically controlled by sustainable heterodyne phase-conjugate physics, accounting for effects which are considered anomalous in terms of standard physics.

References:

[1] Dan Winter: Courses on Sacred Geometry and Coherent Emotion – various essays and seminar registrations at www.soulinvitation.com

[2] Fibonacci Cascades as the "DNA Supra-Code" – Jean-Clode Perez, http://www.goldenmuseum.com/1611GenCode_engl.html

[3] Fast Fourier Transform http://mathworld.wolfram.com/FastFourierTransform.html

[4] Osborne, A. R., Onorato, M., and Serio, M.: The nonlinear dynamics of rogue waves and holes in deep water gravity wave trains, Phys. Lett. A, 275, 386–393, 2000.